8.3 Relative Rates of Growth

Definitions: Faster, Slower, Same-rate Growth

same: $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$

f faster $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$

f slower $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

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$\lim_{x\to\infty} \frac{x^2}{x} = \infty$ x^2 faster than x

$\lim_{x\to\infty} \frac{3x}{x} = 3$ $3x$ grows at same rate as x (same order of magnitude)

$y=3x$

$y=x$

double x, doubles y

$y=3(2x)=6x$

$y=2x$

$y=x^2$

double x

$y=(2x)^2=4x^2$

y is $4x$ as big quadruples

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Which function grows faster?

e^x, x^2

faster

$\lim_{x\to\infty} \frac{e^x}{x^2} = \lim_{x\to\infty} \frac{e^x}{2x} = \lim_{x\to\infty} \frac{e^x}{2} = \infty$

$\ln x, x, x^2$

slower

$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

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$\lim_{x\to\infty} \frac{5^x}{e^x} = \lim_{x\to\infty} \left(\frac{5}{e}\right)^x = \infty$

$y=\ln x$ double x

$y=\ln(2x) = \ln 2 + \ln x$

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Show the functions grow at the same rate

$x, x+\sin x$

$$\lim_{x\to\infty} \frac{x+\sin x}{x} = \lim_{x\to\infty} \frac{1+\cos x}{1} \leq 2$$

$1+1$

same

$\log_a x, \log_b x$

$$\lim_{x\to\infty} \frac{\log_b x}{\log_a x} = \lim_{x\to\infty} \frac{\frac{1}{x\ln b}}{\frac{1}{x\ln a}} = \frac{\ln a}{\ln b} = L$$

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Transitivity of Growing Rates

If $f(x)$ same as $h(x)$

and $h(x)$ same as $g(x)$

then $f(x)$ same as $g(x)$

Show the functions grow at the same rate by comparing both with x

$\sqrt{x^2+5}, (2\sqrt{x}-1)^2$

$$\cancel{\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{(2\sqrt{x}-1)^2}} = \lim_{x\to\infty} \cancel{\frac{2x\cdot\frac{1}{2}(x^2+5)^{-\frac{1}{2}}}{2\cdot\frac{1}{2}x^{-\frac{1}{2}}\cdot 2(2\sqrt{x}-1)^1}}$$

same

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2}} \overset{\text{same}}{=} \lim_{x\to\infty} \sqrt{\frac{x^2+5}{x^2}} = \sqrt{1} = 1$$

$$\lim_{x\to\infty} \frac{(2\sqrt{x}-1)^2}{(\sqrt{x})^2} \overset{\text{same}}{=} \lim_{x\to\infty} \left(\frac{2\sqrt{x}-1}{\sqrt{x}}\right)^2 = 2^2 = 4$$

$\sqrt{x^2} = x$

$(\sqrt{x})^2 = x$

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Sequential vs Binary Searches

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